Stochastic Resonance in Underdamped, Bistable Systems

Rajarshi Ray[a,b] * and Supratim Sengupta[c,d] †

[a] *Tata Institute of Fundamental Research, Homi Bhabha Road, Mumbai 400 005, India.*

[b] *Thoery Group, Saha Institute of Nuclear Physics, 1/AF, Bidhan Nagar Kolkata 700 064 India.*

[c] *Department of Physics and Astronomy, Mcmaster University, Hamilton L8S 4M1, Canada.*

[d] *Theoretical Physics Institute, Department of Physics, University of Alberta, Edmonton T6G 2J1, Canada.*

(February 8, 2008)

Abstract

We carry out a detailed numerical investigation of stochastic resonance in underdamped systems in the non-perturbative regime. We point out that an important distinction between stochastic resonance in overdamped and underdamped systems lies in the lack of dependence of the amplitude of the noise-averaged trajectory on the noise strength, in the latter case. We provide qualitative explanations for the observed behavior and show that signatures such as the initial decay and long-time oscillatory behaviour of the temporal correlation function and peaks in the noise and phase averaged power spectral density, clearly indicate the manifestation of resonant behaviour in noisy, underdamped bistable systems in the weak to moderate noise regime.

PACS numbers: 02.50.Ey, 05.10.Gg, 05.40.Ca

*E-mail: rajarshi.ray@saha.ac.in

†E-mail: sengupta@physics.mcmaster.ca

I. INTRODUCTION

Noise usually has a disruptive role in nature. However, there is a phenomenon [1,2] in which noise helps in making the system behave in a more coherent manner. This intriguing and rather counterintuitive phenomenon called *Stochastic Resonance*(SR) [3] was first proposed as an explanantion of the observed periodicity in the ice ages on earth [1,2], but has since been observed in a large variety of physical, chemical and biological systems [4]. Bistable systems subject to noise and periodic modulation constitute the most common class of systems which exhibit SR.

The phenomenon of SR can be thought of as resulting from the synchronization of a stochastic time-scale (determined by the thermal transition rate over the barrier) and a deteministic time scale (determined by the time-scale of periodic modulation). When the noise strength is appropriately tuned to ensure the synchronization of the these two time scales, the system exhibits stochastic resonance. SR manifests itself through a variety of signatures such as peak in the amplitude of the noise-averaged trajectory versus noise strength, peak in the distribution of peak heights in the residence time distribution (RTD) [5–7], peaks in the power spectral density (or Signal-to-Noise Ratio) [8] at the modulating frequency and late-time oscillatory behaviour of the temporal auto-correlation function [5]. SR has been extensively investigated in many overdamped systems, but it has'nt been explored in great detail in underdamped systems. (references listed in [5,9] are some exceptions). Apart from the intrinsic interest in exploring in detail the manifestations of SR in the underdamped regime, there are other motivations for focussing on this regime. In particular, relativistic systems are governed by underdamped equations of motion and investigating SR in single particle underdamped systems may provide valuable insights into the manifestations of SR in relativistic field theories.

We will focus on numerically investigating the phenomenon of SR in underdamped bistable systems subject to white noise and periodic modulation. The system we are dealing with can be described by the Langevin equation

$$\frac{d^2x}{dt^2} + \eta\frac{dx}{dt} - x + x^3 = A_o cos(\omega t + \theta) + \xi(t)$$
$$\langle \xi(t)\xi(t')\rangle = 2T\eta\delta(t-t') \qquad (1)$$

where T is the temperature of the background heat bath, η is the dissipation coefficient, A_0 and ω are the amplitude and frequency of periodic modulation repectively. ξ is a Gaussian distributed (delta correlated) white noise term which satisfies the fluctuation-dissipation theorem. For simplicity, we have made use of scaled, dimensionless quatities in the above equation. The potential governing particle dynamics is of standard double well type, modulated by a periodic signal with small amplitude.

$$V(x,t) = -\frac{1}{2}x^2 + \frac{1}{4}x^4 - A_0 x\cos(\omega t + \theta) \quad (2)$$

In the absence of periodic modulation, the doubly degenerate minima of the double well potential are located at $x_m = \pm 1$ and the barrier height is given by $\Delta V_0 = 0.25$ The modulation amplitude A_0 is small enough to ensure the bistability of the potential at all times. Moreover, the noise strength, $D = T\eta$ (for the underdamped case) is much smaller than the barrier height in the absence of modulation i.e. $D << \Delta V_0$. We will also be working in the moderately weak noise regime $D \lesssim A_0$. There has been several analytical studies of SR in overdamped systems [3], but those investigations were based on perturbation theory and therefore restricted to the perturbative regime $A_0 << D$ or the regime of validity of the adibatic approximation $A_0 <<$

$D << \Delta V_0$. An analytical study of underdamped, periodically modulated systems was carried out by Jung and Hanggi [10], based on the solution of the 2-dimensional and 3-dimensional Fokker-Planck equations for probability distribution. They obtained the asymptotic probability distribution and showed that for uniformly distributed initial phases, the power spectrum exhibits delta-functions spikes at multiples of the driving frequency. Previous experimental work on SR in underdamped systems [5] have been restricted to the strong noise regime characterized by $T\eta >> \Delta V$ and focussed primarily on signatures like the signal-to-noise (SNR) ratio and residence time distribution. More recently, Alfonsi et.al. [9] have shown that that an intra-well SR can coexsist with conventional inter-well SR in underdamped systems and is manifest as twin peaks in the plot of power spectral density versus noise strength. Their paper further extended the work of Stocks et.al. dealing with SR in monostable systems [11] using methods of Linear Response Theory. Evidence of SR in *overdamped* bistable systems without explicit symmetry breaking and subject to periodically oscillating noise strength have been discussed in [12,13].

Our main aim in this paper is to numerically explore the various manifestations of SR in *underdamped* systems in the moderately weak noise and moderate forcing regime. This is distinct from the strong noise, weak forcing regime [5] and strong forcing regime [9] explored in earlier works. Although the effect of weak noise was considered in the work of Alfonsi *et.al.* [9], their analysis was focussed on the behaviour of the power spectrum and in distinguishing between intrawell and interwell SR in the high frequency forcing regime. We point out an unusual behaviour of the noise-averaged trajectory, in under-damped systems in the moderate-weak noise and weak forcing regime, which is quite distinct from its behaviour in over-damped systems where it is widely used as a characteristic signature of SR. We find that there is no increase in the amplitude of noise averaged trajectory with increasing noise strength, unlike SR in overdamped systems. This important distinction in the dynamics of overdamped and underdamped systems was not recognized earlier. It can be attributed to the delicate balancing act between noise and dissipation in underdamped systems, which also reduces the window size in parameter space for observing SR-like behaviour. Our results supplement the analysis of SR in underdamped systems carried out earlier [5,9].

The paper is organized as follows. In Sec. II, we carry out a detailed numerical study of SR in underdamped systems in the weak noise, moderate forcing regime. The effect of changing system parameters on the various distinct signatures of SR are described in detail. We conclude with a brief summary and discussion of our results in section III.

II. STOCHASTIC RESONANCE IN UNDERDAMPED SYSTEMS : MODERATE AND WEAK NOISE REGIME

The key idea behind SR is that the synchronization of the stochastic and deterministic time scales of the system can result in a more coherent response of the system even on *increasing* the noise strength, for a small range of values of the modulating frequency. Even though SR was first investigated for overdamped systems, this intuitive notion of SR was also found to be valid for underdamped systems [5]. The interesting area of parameter space is such that neither the periodic modulation of the potential, nor the thermal noise activation is individually capable of ensuring periodicity in the inter-well hopping of the particle. For large

noise strengths i.e. $D >> \Delta V_0$ the particle has a large enough thermal energy to easily surmount the potential barrier and therefore does not feel the presence of the barrier. In the absence of coupling to a heat bath, the small amplitude of modulation, $A_0 < \Delta V_0$ and forcing frequency $\omega << \tau_r^{-1}$ (where τ_r is the intra-well relaxation time scale) cannot provide sufficient energy to the particle to surmount the barrier. However, both these effects acting in conjunction can, under certain circumstances, lead to periodic behaviour in the noise-averaged particle trajectory. In general, the thermal transition rate (Kramer's rate) over the barrier $\tau_k^{-1} \equiv r_k$, depends on both the noise strength and dissipation coefficient. The time scale of periodic modulation ($\tau_0 = \frac{2\pi}{\omega}$) is determined by the forcing frequency ω. Due to the odd power of the coupling between the particle position and the periodic forcing, the symmetry of the potential is explicitly broken. The particle in either well encounters the smallest (largest) barrier twice during a single period. If $\tau_k^{(-)}$ ($\tau_k^{(+)}$) are the corresponding thermal transition time-scales, the noise averaged particle trajectory becomes periodic when $\tau_k^{(-)} < \tau_0 < \tau_k^{(+)}$. Inter-well hopping is therefore synchronized when the following (approximate) condition between the deterministic and stochastic time scales is satisfied $\tau_k \simeq \frac{\tau_0}{2} \Rightarrow \omega_{sr} \sim \frac{\pi}{\tau_k}$. Fox and Lu [14] have shown that this matching condition which yields the corresponding resonant value of the noise strength ($D_{sr} = (T\eta)_{sr}$), is an approximate one and does not become exact in any limit.

At this stage it is necessary to point out the crucial difference between our results and those obtained via analog simulations [5] in the work of Gammaitoni et.al. The main difference lies in the distinctly different regime of study undertaken in this paper. Even though the work of Gammaitoni et.al. [5] dealt with investigating SR in underdamped systems, their choice of parameters restricted their study to the strong noise regime characterized by the condition $T\eta > \Delta V_0$. However, we decided to explore the weak-to-moderate noise regime characterized by the condition $T\eta << \Delta V_0$ and $T\eta \lesssim A_0$. Moreover, the validity of their theoretical analysis depended on the applicabiliy of perturbation theory, which amounts to the condition $A_0 << T\eta$ implying that the influence of modulation is small compared to thermal activation. This lead to the temperature dependence of the amplitude of the noise-averaged trajectory. However, our regime of study is not amenable to a perturbative analysis since in our case $T\eta \lesssim A_0$. Moreover, $A_0 \lesssim \Delta V_0$ further implying a substantial (non-perturbative) modulation of potential by the driving force, even though the modulation is insufficient to destroy the bistability of the potential at any time and is therefore distinct from regime where resonant trapping [15] is observed. We find that this leads to an important difference in the behaviour of the noise averaged trajectory characterized by the lack of dependence of the amplitude with noise strength. This is distinct from the manifestation of SR in the overdamped case and in the underdamped case studied earlier [5].

The particle dynamics in the moderate-strong damping regime is dominated by fluctuations and the Kramer's rate in this regime is given by

$$r_k = \frac{\sqrt{1+\eta^2/4} - \eta/2}{\sqrt{2}\pi} e^{-\beta \Delta V_0} \tag{3}$$

However, this expression is derived under the assumption that the intrawell relaxation time-scale of a particle is much smaller than the thermal transition time scale and the particles are able to thermalize before escaping over the barrier. As the dissipation coefficient decreases, this assumption breaks down as the motion of the particles starts being domi-

nated by energy diffusion. Appropriate modification to the rate formula in the very weak dissipation regime was obtained by Kramers [16,17], Carmeli *et.al.* [18] and by Buttiker *et.al.* [19], in which the transition rate was found to be proportional to η

$$r_k = \frac{\eta\beta I(E_0)}{2\sqrt{2}\pi} e^{-\beta\Delta V_0} \tag{4}$$

where $I(E_0)$ is the action at the top of the barrier. [Carmeli *et.al.* [18] and Buttiker *et.al.* [19] also obtained appropriate corrections to the above formula by relaxing the approximation that the distribution function drops to zero at the top of the barrier, a crucial assumption in the derivation of Eq.(4).] The approximate resonant frequency $\omega_{sr} \sim \frac{\pi}{\tau_k}$ for underdamped systems has to be estimated with this modified rate formula in the very weak dissipation regime. In our work, ω_{sr} was used only to identify the regime for resonant behaviour in underdamped systems.

We numerically solved Eq.(1)using a second order stochastic Runge-Kutta algorithm. The noise averaged trajectory was obtained by averaging over one hundred noise realizations. The upper plot of Fig.1 shows the noise averaged trajectory for two different values of bath temperature, obtained by solving the overdamped Langevin equation given by

$$\frac{dx}{dt} - x + x^3 = A_o cos(\omega t) + \xi(t)$$
$$\langle \xi(t)\xi(t') \rangle = 2T\delta(t - t') \tag{5}$$

A clear dependence on the amplitude of the oscillation on temperature is observed. The maximum of the amplitude corresponds to the temperature that satisfies the resonant condition $\omega_{sr} \sim \frac{\pi}{\tau_k}$. When the temperature increases beyond 0.08, the amplitude decreases and eventually the dynamics becomes noise dominated resulting in loss of periodicity. In contrast, the lower plot of Fig.1 shows the noise averaged trajectories for underdamped case obtained by solving Eq.(1). The trajectories for the two different temperature values almost overlap indicating a lack of dependence on the temperature of the heat bath. This important aspect of SR in underdamped system in the weak-moderate noise regime was not realized earlier. It can be attributed to the fact that the noise strength $(T\eta)$ in underdamped systems depend not only on the temperature of the heat bath, but also on the dissipation coefficient η. Since η is very small for underdamped systems, an increase in temperature is offset by the smallness of η which therefore does not lead to any substantial increase in noise strength.

A. Residence Time Distribution

An useful characterization of SR [5] is obtained by considering the discrete stochastic process associated with the barrier crossing time. The residence time then corresponds to the time spent by the particle in a well between successive transitions over the barrier and is obtained by taking the difference between successive barrier crossing times. The residence time was obtained numerically by noting the time required by the particle starting from either well to cross a point specified on the other side of the barrier for each trajectory. The average of these times over all realizations yielded the mean residence time. The choice of the crossing point, although arbitrary, was made to ensure that the particle had indeed made a transition from one well to another. This rules out trajectories where the particle just crosses the barrier, lingers around the top of the barrier, before falling back into the well it started from. This phenomenon is particularly relevant in underdamped systems and would lead to underestimating the residence time. We emphasize that the two-state approximation was not been made in estimating the residence time.

Although it is often customary to make such an approxiation in analyzing hopping dynamics in overdamped systems, it would lead to wrong estimates of the residence time for reasons mentioned above. Fig.2 shows the residence time distribution for 3 different values of the bath temperature. A pattern of peaks in the RTD with exponentially decaying peak heights is observed. (The peak height at $t/\tau = 0.5$ have been normalized to unity in Figs.2,4,6.) The likelihood of the particle hopping over the barrier is largest when the barrier height to be surmounted is the least. Due to the modulation of the potential, the particle encounters the smallest barrier twice during every modulation period. When perfect synchronization is achieved i.e. the resonant condition is satisfied, the particle hops over the barrier *twice* every modulation period and a single large peak in the RTD is observed at $t = 0.5\tau_0$. When $T > T_{sr}$ (where T_{sr} is the bath temperature which corresponds to the resonant frequency ω_{sr} for fixed η), noisy dynamics starts dominating and synchronization of inter-well hopping is lost. The inter-well transitions occur for $t < 0.5\tau_0$, which accounts for a sharp peak in RTD for $t < 0.5\tau_0$. For $T < T_{sr}$, perfect synchronization is lost because the thermal transition time scale(τ_k) is larger than $0.5\tau_0$. If the particle in unable to traverse the barrier when the barrier height is minimum, it has to wait for one full period before it again faces the smallest barrier. This accounts for the multiple peaks in the RTD at *odd* multiples of $0.5\tau_0$.

We would like to emphasize that the role of dissipation on SR can only be addressed by studying UD systems. This is easily understood since the thermal transition time scale for inter-well hopping depends on the η, albeit weakly. However, the signature of SR associated with the periodicity of the noise-averaged trajectory is not very sensitive to changes in η and ω. This can be seen from the plots of the noise averaged trajectory for different values of η and ω as shown in Fig.3 and Fig.5. The periodicity in the noise averaged trajectory is observed even when η is increased by about two orders of magnitude. A more sensitive indicator of resonant behaviour is the residence time distribution. Nevertheless, the dependence on η is rather weak as is evident from the plots for the RTD shown in Fig.4. Increasing η increases the noise strength thereby increasing fluctuations, but that increase is balanced out by the damping of fluctuations resulting from a larger η. Because of this delicate balance between fluctuation and dissipation observed in underdamped systems, the dependence of RTD on η is weaker than its dependence on the bath themperature. This is evident from Fig.4 which shows plots of the RTD for three different values of η. For very small dissipation, τ_k becomes larger than $0.5\tau_0$ and this leads to loss of synchronization of inter-well transitions resulting in multiple peaks in RTD at odd multiples of $0.5\tau_0$. However, the weak dependence on η does not affect the shape of the RTD on increasing η by an order of magnitude (compared to the benchmark value of $\eta = 0.03$) as is evident from the last two plots of Fig.4.

Fig.5. shows the the noise averaged trajectory for three different values of the modulating frequency. The periodic behaviour is observed even for values of ω substantially different from the value which satisfies the resonant condition. However, as ω is increased, synchronization is lost and the dynamics becomes noise dominated as is evident from the last plot of Fig.5. However the sensitivity to change in ω is more pronounced in the plots for the RTD shown in Fig.6. When perfect synchronization is attained, inter-well hopping over the barrier occurs when the the barrier height is minimum. This amounts for a single sharp peak in the RTD at $t = 0.5\tau_0$ corresponding to the resonant frequency $\omega_{sr} = 0.03$, as seen in the first

plot of Fig.6. For larger (or smaller) modulation frequencies, synchronizations is lost and this is manifest through appearance of multiple peaks (with exponentially decreasing peak heights) in the RTD at odd multiples of the half-period of modulation.

However, a proper signature of SR based on RTD is characterized by a peak in the distribution of peak heights (at half the driving period) versus noise [6]. We also find a peak around T_{sr}, in the distribution of peak heights plotted against temperature. However, no such peak is observed when the distribution of peak heights are plotted against the dissipation coefficient, providing another indication that the hopping dynamics is less sensitive to dissipation coefficeint in the weak damping regime. This characterization of SR has lead to some controversy regarding the identification of SR as a bonafide resonance [7,20–22] in the overdamped case. Choi et.al. [7] have argued that for large A_0/D values, peaks in the RTD distribution were insufficient to imply resonant behaviour in a practically viable frequency range. Our aim in carrying out the RTD analysis for varying temperature, dissipation and driving frequency was to indicate that the behaviour of the RTD in underdamped systems follows the same pattern seen in overdamped systems and can be understood in terms of synchronization of hopping over the barrier. However, since we are working in the large A_0/D regime, RTD alone cannot be used to infer about the manifestation of SR in underdamped systems. In order to unambiguously show that SR is observed in underdamped systems in the weak and moderately weak noise regime (i.e.$A_0/(T\eta) >> 1$) investigated in this paper, we present below two other signatures of SR.

B. Auto-correlation Function and Power Spectrum

Evidence for SR can also be obtained by studying the behaviour of the auto-correlation function (ACF) [5], defined by the equation

$$C(t) = \langle x(t)x(0)\rangle \tag{6}$$

where the $\langle ... \rangle$ implies averaging over noise realizations. The first plot in Fig.7 shows the early-time exponential decay of the ACF for four different values of the bath temperature. Larger bath temperatures lead to a faster decay in the ACF, $C(t)$, since large fluctuations destroy correlations at large temporal separations. The initial decay of the ACF is followed by oscillatory behaviour with a frequency equal to that of the the modulation frequency. Such a behaviour is symptomatic of SR in driven, diffusive systems subject to thermal noise. The lower graph of Fig.7 shows a distribution of the temporal ACF at late times which indicates a clear oscillatory dynamics. The gray solid line is a sinosoidal fit to the numerical ACF data and has a frequency equal to the modulation frequency of the system. Note that our choice of parameters does not allow for a perturbative analysis which was carried out in [5] to obtain anaytical expressions for the amplitude of the noise averaged trajectory and the auto-correlation function. Such an approximation is only valid for small perturbations $A_0 x_m << T$. [1] However, in our case, $T \gtrsim \Delta V_0$. Furthermore, our regime of study lies *beyond* the perturbative regime, since $A_0 x_m \lesssim T$.

Another common signature of SR can be obtained by looking at the noise and phase

[1] In the notation of [5], D corresponds to the temperature (T) of the heat bath.

averaged power spectral density $S(\omega)$ obtained by taking the fourier transform of the ACF $C(t)$.

$$S(\omega) = \int_{-\infty}^{\infty} \exp^{-i\omega t} << x(t+\tau)x(t) >> d\tau \tag{7}$$

where the inner brackets indicate avergaing over noise realizations and the outer brackets denote avergaing over the initial random phase. We averaged over 50 different noise realizations and 20 different values of random initial phase to first obtain the noise and phase averaged temporal ACF for 64 different values of temporal separation. The fourier transform of the noise and phase averaged ACF then yielded the noise and phase averaged power spectral density. Fig.8 shows the power spectral density as a function of the frequency, obtained for four different values of the bath temperature, all other parameters remaining fixed. The plots clearly show a sharp peak at the the modulation frequency, and indicates that the peak strength at the driving frequency goes through a maximum as a function of the noise strength. Although, the exact location of the maxima is difficult to ascertain precisely, it is nevertheless clear that the maxima is close to $T = 0.2$, which is the temperature at which the synchronization condition is satisfied for the given choice of parameters. These additional signatures clearly establish that the observed synchronization, in the weak and moderately weak noise regime of underdamped bistable systems, is indeed a manifestation of stochastic resonance.

III. CONCLUSIONS

To summarize, we have carried out a numerical study of SR in underdamped, bistable systems in the regime characterized by the conditions $T\eta << \Delta V_0$ and $T\eta << A_0 x_m$. This is distinct from the regime investigated earlier [5] and is not amenable to an analysis based on treating the modulation of the potential as a small perturbation. A comparison between overdamped and underdamped dynamics was carried out. No significant dependence of the amplitude of the noise averaged-trajectory, on the temperature of the heat bath, was found in the underdamped scenario. This is in marked contrast to the the overdamped case, where the amplitude depends sensitively on the bath temperature. Synchronization of hopping with periodic modulation of the potential well was observed in the behavior of the RTD, as in the overdamped case. The RTD was found to be more sensitive to changes in bath temperature and only weakly dependent on changes in the dissipation coefficient. The autocorrelation function and the power spectral density were also obtained for various values of the bath temperature to unambiguously confirm that the underdamped system does indeed exhibit resonant behaviour in the weak noise and weak forcing regime. The power spectrum was characterized by a sharp peak at a value close to that of the modulating frequency, with the largest peak height corresponding to the temperature at which the resonant condition was satisfied. These signatures provide evidence that SR in underdamped systems depends on the interplay between the bath temperature and the dissipation coefficient and leads to a rather distinct pattern of dynamics, not observed in the previously investigated regimes of parameter space.

An interesting extension of this work would involve looking for evidence of SR in relativistic field theoretic models. The study of stochastic resonance for spatially extended systems has been carried out for Ginzburg-Landau type field theories [23], but has been restricted to the over-damped regime .There it was found that an appropriate choice of the

frequency of the periodic driving obtained by matching the thermal activation time-scale to half the period of the modulating background, can result in periodically synchronized behavior of the mean field about $\phi = 0$ (see Fig.2 of Ref. [23]). It would be interesting to explore the possibility of observing SR in *underdamped*, field-theoretic models exhibiting spontaneous symmetry breaking of a Z_2 symmetry since it would provide the first evidence of emergence of coherent behaviour in noisy, underdamped field theory.

IV. ACKNOWLEDGEMENTS

We would like to thank A.M. Srivastava for useful discussions and encouragement. SS would also like to thank F.C. Khanna for support and encouragement. The work of SS was funded in part by NSERC, Canada.

REFERENCES

[1] R. Benzi, A. Sutera and A. Vulpiani, J. Phys. **A18**, 2239 (1985); R. Benzi, G. Parisi, A. Sutera and A. Vulpiani, Tellus **34**, 10 (1982); R. Benzi, A. Sutera, G. Parisi and A. Vulpiani, SIAM J. Appl. Math. **43**, 565 (1983).

[2] C. Nicolis and G. Nicolis, Tellus **33**, 225 (1981).

[3] P. Jung, Phys. Rept. **234**, 175 (1993); L. Gammaitoni, P. Hanggi, P. Jung and F. Marchesoni, "Stochastic Resonance", Rev. Mod. Phys. **70**, 223 (1998).

[4] F. Moss,*Stochastic Resonance : From Ice Ages to the Monkey's Ear*, in *Contemporary Problems in Statistical Physics*, ed. G.H. Weiss (SIAM, Philadelphia 1994), pp.205-253.

[5] L. Gammaitoni, F. Marchesoni, E. Menichella-Saetta and S. Santucci, Phys. Rev. Lett. **62**, 349 (1989); L. Gammaitoni, E. Menichella-Saetta, S. Santucci, F. Marchesoni and C. Presilla, Phys. Rev. **A40**, 2114 (1989).

[6] T.Zhou, F. Moss and P. Jung, Phys. Rev. **A42**, 3161 (1990)

[7] M.H. Choi, R.F. Fox and P. Jung, Phys. Rev. **E57**, 6335 (1998).

[8] B. McNamara, K. Weisenfeld and R. Roy, Phys. Rev. Lett. **60**, 2626 (1988); G. Vemuri and R. Roy, Phys. Rev. **A39**, 4668 (1989).

[9] L. Alfonsi, L. Gammaitoni, S. Santucci and A. R. Bulsara, Phys. Rev. **E62**, 299 (2000).

[10] P. Jung and P. Hanggi, Phys. Rev. **A41**, 2977 (1993).

[11] N. G. Stocks, N. D. Stein, S.M. Soskin and P.V.E. McClintock, J. Phys. **A25**, L1119 (1992).

[12] A. Neiman and L. Schimansky-Geier, Phys. Rev. Lett. **72**, 2988 (1993).

[13] L. Gammaitoni, F. Marchesoni and S. Santucci, Phys. Lett. **A195**, 116 (1994)

[14] R.F. Fox and Y. Lu, Phys. Rev. **E48**, 3390 (1993).

[15] F. Apostolico, L. Gammaitoni, F. Marchesoni and S. Santucci, Phys. Rev. **E55**, 36 (1997).

[16] H. A. Kramers, Physica (Utrecht) **7**, 284 (1940).

[17] P.Hanggi, P. Talkner and M. Borkovec, Rev. Mod. Phys. **62**, 251 (1990).

[18] B. Carmeli and A. Nitzan, Phys. Rev. Lett. **49**, 423 (1982).

[19] M. Buttiker, E.P. Harris and R. Landauer, Phys. Rev. **B28**, 1268 (1983).

[20] L. Gammaitoni, F. Marchesoni and S. Santucci, Phys. Rev. Lett. **74**, 1052 (1995).

[21] D. Dan and A.M. Jayannavar, cond-mat/0303471.

[22] K. Sekimoto, J. Phys. Soc. Jpn. **66**, 6335 (1997); F. Takagi and T. Hondou, Phys. Rev. **E60**, 4954 (1999).

[23] R. Benzi, A. Sutera and A. Vulpiani, J. Phys. **A18**, 2239 (1985).

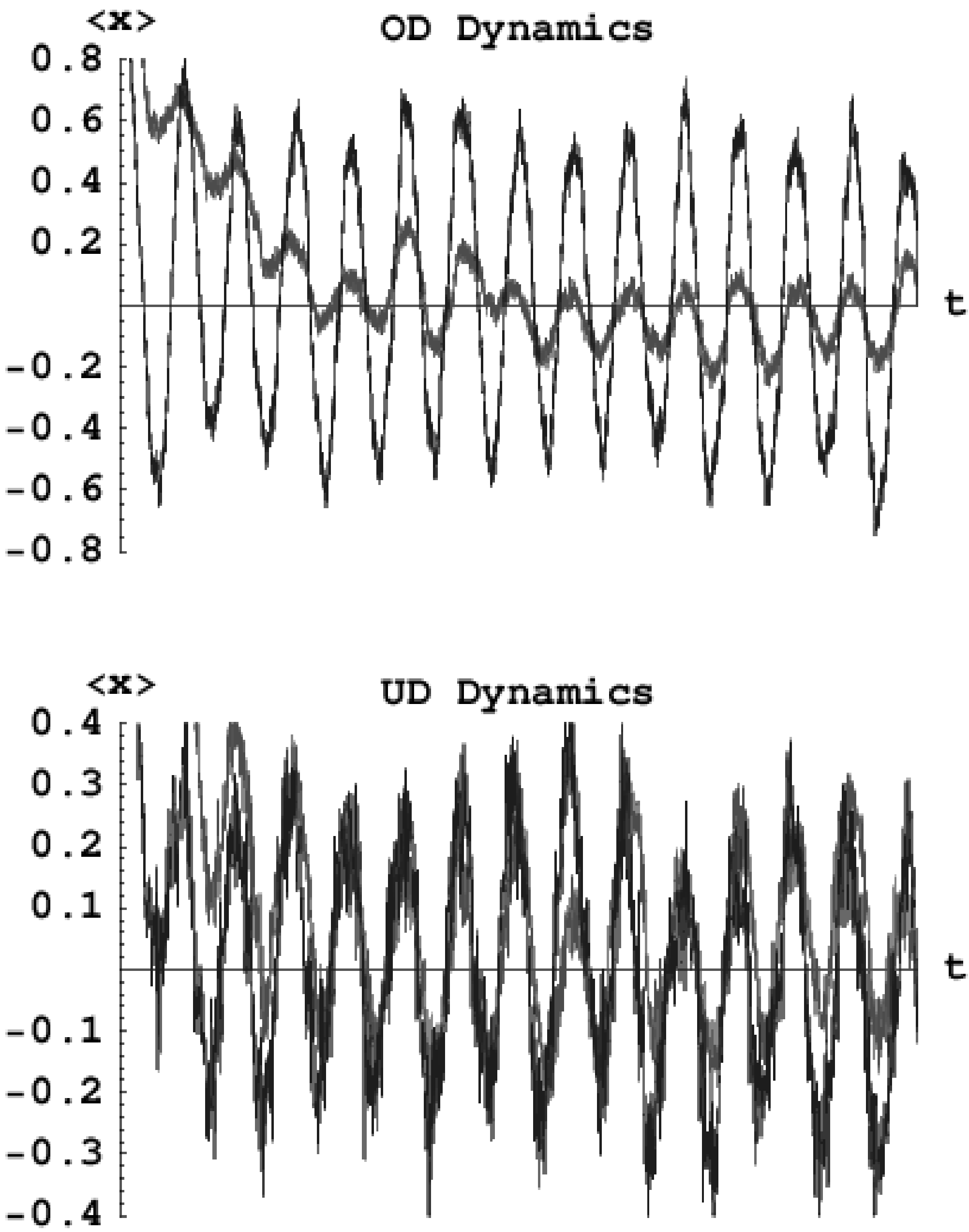

FIG. 1. Plots showing the noise averaged trajectory with two different temperatures for the overdamped case (top) and underdamped case (bottom).The bath temperatures for the two trajectories shown correspond to $T = 0.04, 0.08$ (top) and $T = 0.08, 0.2$ (bottom).

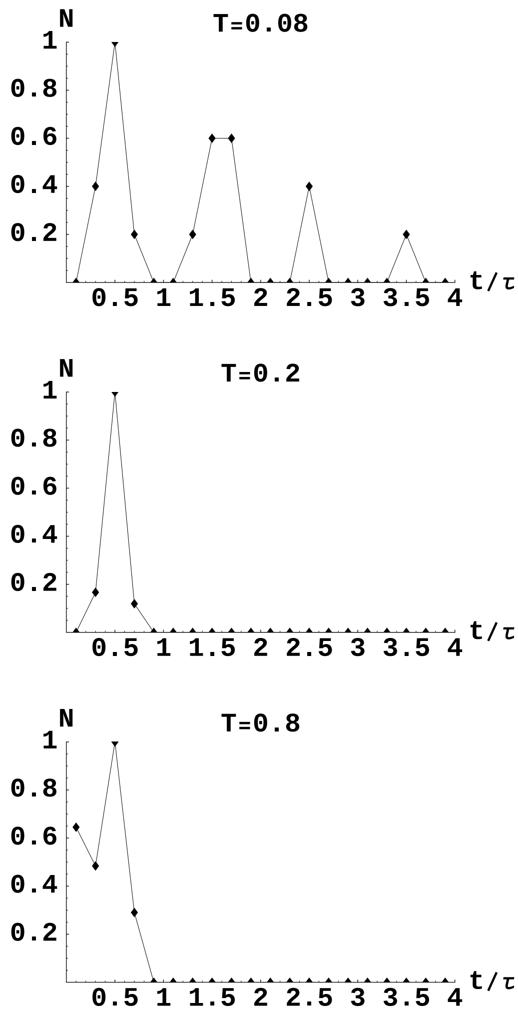

FIG. 2. Residence Time Distribution for three different values of bath temperature. Other fixed parameter values are $\eta = 0.03, A_0 = 0.1, \omega = 0.03$. The temperature at which the resonance condition is satisfied for fixed ω and η is $T \equiv T_{sr} = 0.2$

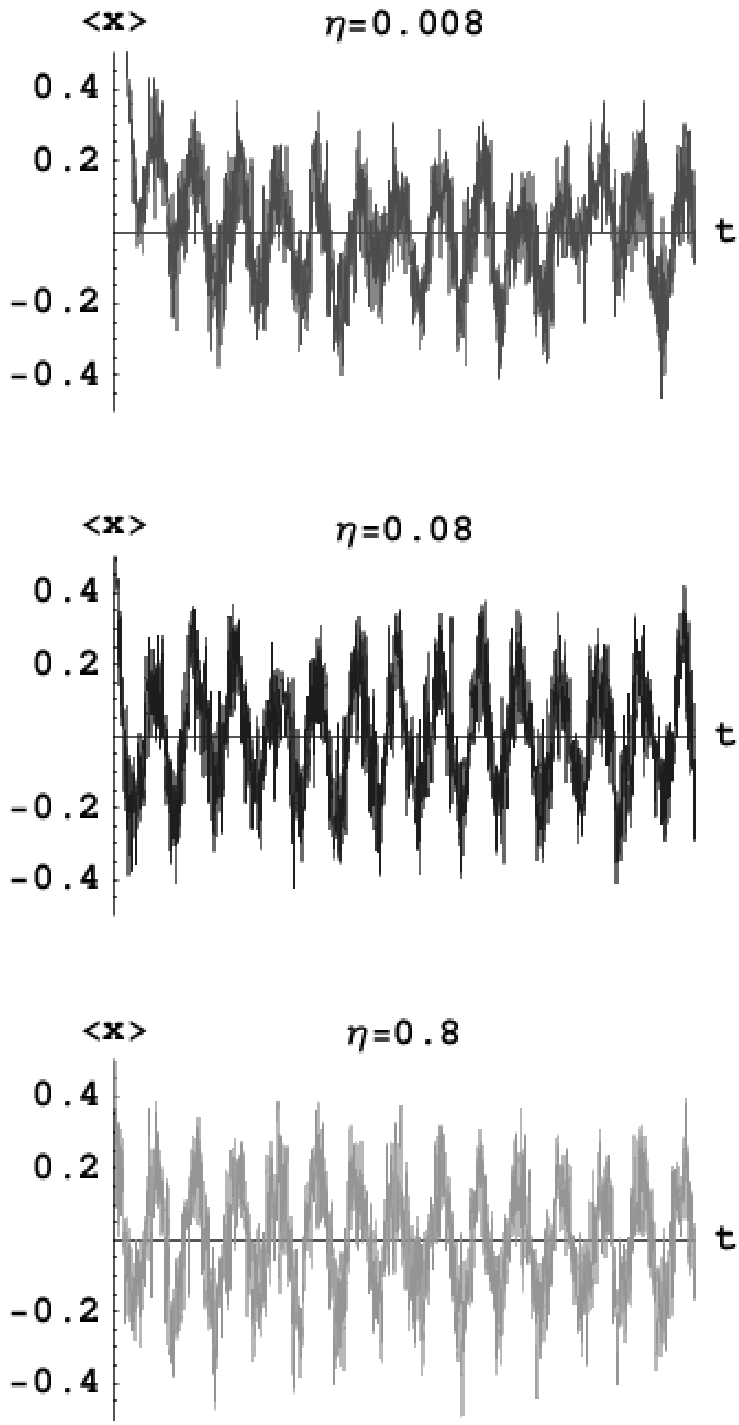

FIG. 3. Noise averaged trajectory for three different values of η. Other fixed parameter values are $T = 0.2, A_0 = 0.1, \omega = 0.03$. The dissipation at which the resonance condition is satisfied for the given ω and T value is $\eta \equiv \eta_{sr} = 0.03$

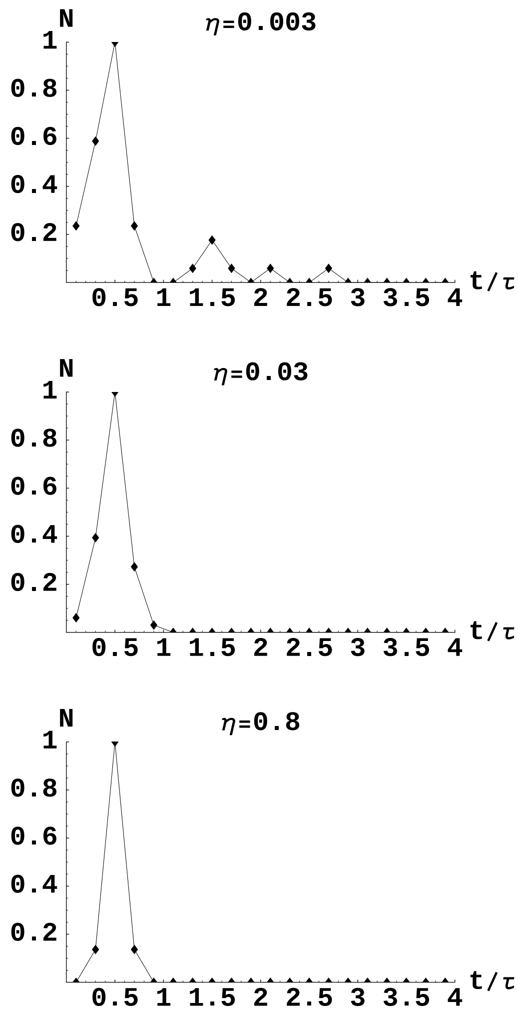

FIG. 4. Residence Time Distribution for three different values of η. Other fixed parameter values are $T = 0.2, A_0 = 0.1, \omega = 0.03$.

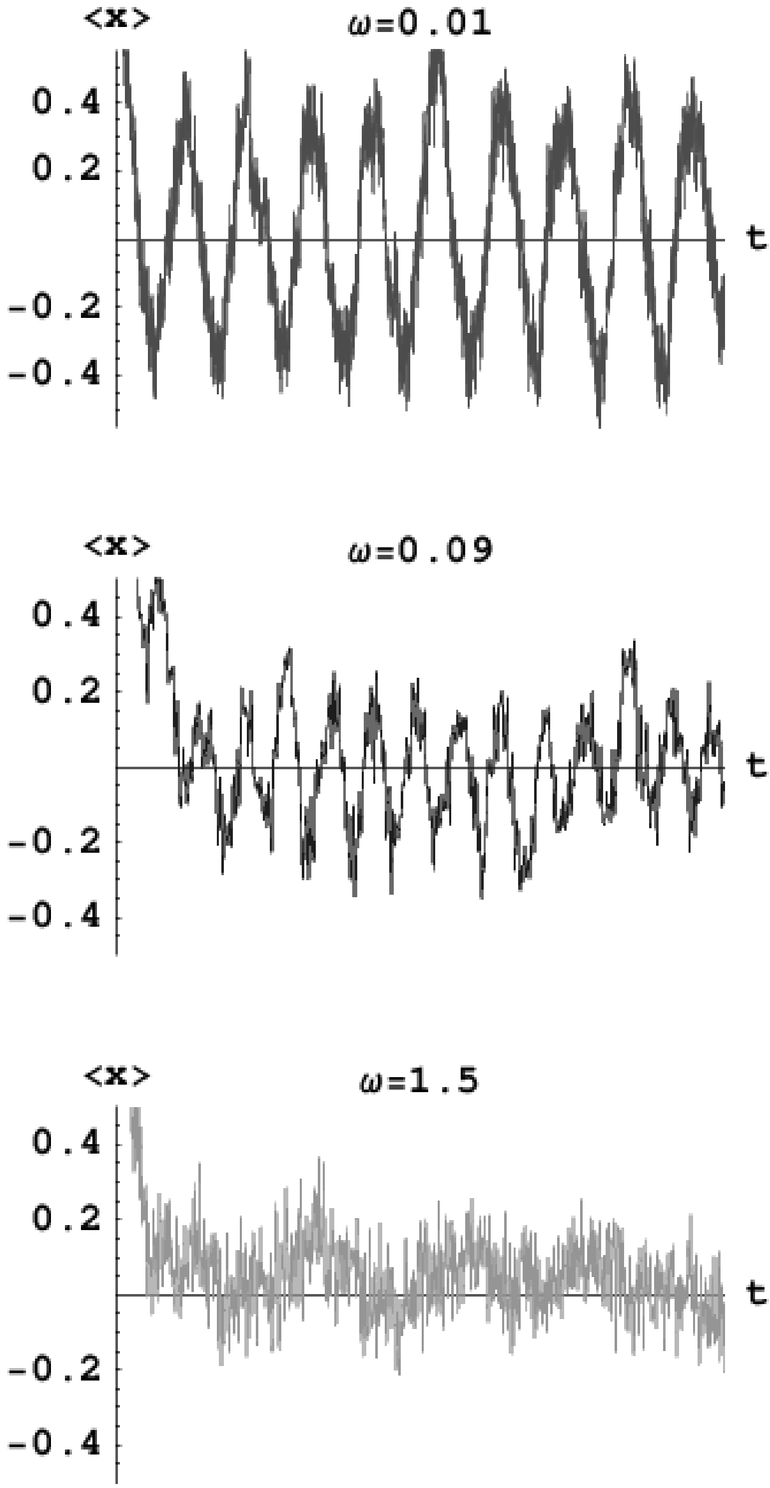

FIG. 5. Noise averaged trajectory for three different values of the modulating frequency. Other fixed parameter values are $T = 0.2, A_0 = 0.1, \eta = 0.03$. The resonant frequency for the given η and T values correspond to $\omega \equiv \omega_{sr} = 0.03$

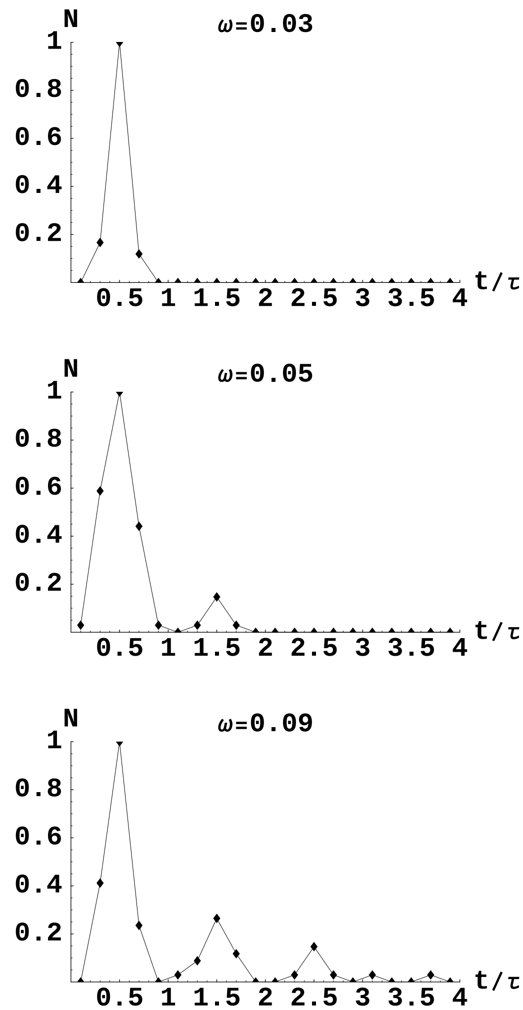

FIG. 6. Residence Time Distribution for three different values of the modulating frequency. Other fixed parameter values are $T = 0.2, A_0 = 0.1, \eta = 0.03$. The resonant frequency for the given η and T values correspond to $\omega \equiv \omega_{sr} = 0.03$

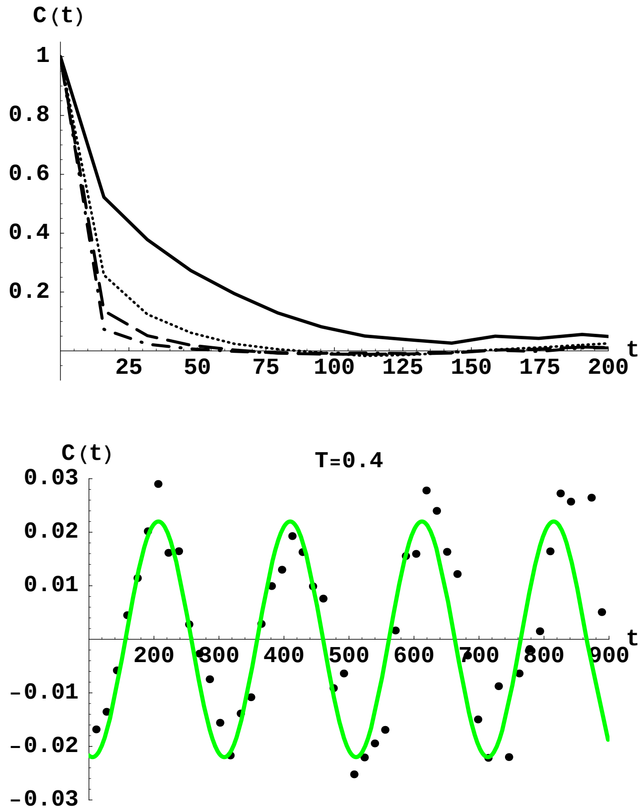

FIG. 7. The upper plot shows the evolution of the temporal auto-correlation function (ACF) at early times and clearly indicates the exponential decay of the ACF. The four curves correspond to $T = 0.2, 0.4, 0.6, 0.8$ respectively from top to bottom. The lower plot shows the oscillatory behaviour of the ACF at late times for T=0.4. The solid line is a sinosoidal fit to the numerical data with an oscillating frequency $\omega = 0.03$. Other parameter values are $\eta = 0.03, \omega = 0.03, A_0 = 0.1$.

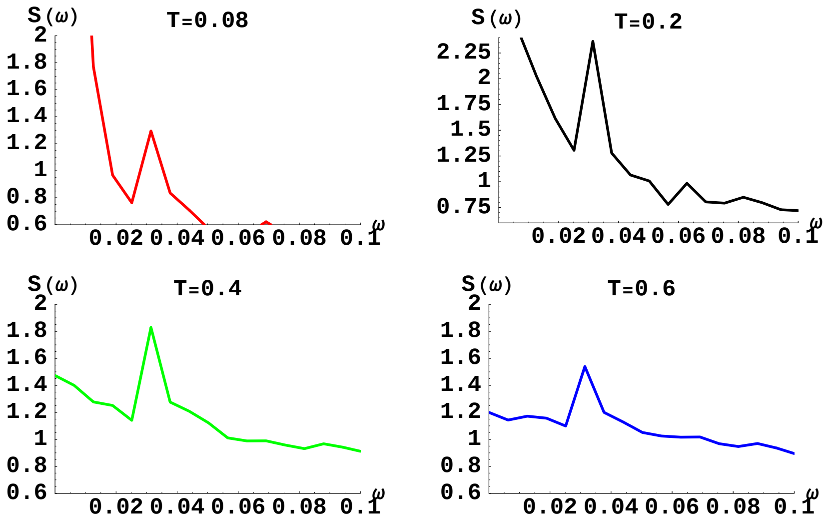

FIG. 8. Plots of the power spectrum showing peaks at the resonant frequency $\omega_{sr} = 0.03$. The largest peak is seen for $T = T_{sr} = 0.2$ which corresponds to the temperature at which the resonance condition is satisfied for $\omega = 0.03, \eta = 0.03, A_0 = 0.1$.